Transient Thermal Sensitivity Analysis During Solar Eclipse with Discontinuous Heat Load

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Nomenclature

A = total surface area of the nodal element exposed to solar radiation, m^2
A_c = cross-sectional area of the nodal element, m^2
A_s = surface area of the nodal element, m^2
C = diagonal matrix whose components C_i, $i = 1, 2, \ldots, N$, represent thermal capacitance of the nodes [Eq. (2)], W/K
$F_{i,j}$ = geometrical view factor between the ith and jth nodes
$\boldsymbol{f}$ = row vector with components $f_1, f_2, \ldots, f_N$, W
f_i = functions defined in Eq. (2), $i = 1, 2, \ldots, N$, W
G_i = functional form of the temperature constraint at the ith node [Eqs. (4) and (5)], K
$g_i(t)$ = heat generated at the ith node [Eq. (1)], W
$K_{i,j}$ = thermal conductivity of the material between ith and jth node, W/m K
$l_{i,j}$ = linear elemental length between ith and jth nodes, m
M = total number of design parameters considered
N = total number of nodes in the system
$S_i(\beta_J)$ = temperature sensitivity of the ith node with respect to parameter β_J, K/unit of β_J
s = solar intensity, W/m^2
$\boldsymbol{T}$ = row vector with components $T_1, T_2, T_3, \ldots, T_N$, K
T_i = temperature of the ith node, K
$T_{\max}$ = maximum permissible temperature of the shunt regulator, K
$T_{\min}$ = minimum permissible temperature of the shunt regulator, K
t = time, s
t_f = terminal time in the integration [Eq. (6)], s
α = solar absorptance of the surface of the shunt regulator
β_J = Jth design parameter, $J = 1, 2, \ldots, M$, unit of the parameter considered
ε = infrared emittance of the surface of the shunt regulator
θ = angle between normal to any surface and the sun vector, rad
λ = constant used in the regularization of discontinuity (Fig. 1), s
σ = Stefan–Boltzmann constant, W/m^2 K^4
φ = row vector whose components $\varphi_1, \varphi_2, \ldots, \varphi_N$ are Lagrange multipliers [Eq. (6)]

Subscripts

i, j = ith and jth nodes
sp = three-dimensional space

Superscript

l = transpose of a matrix

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Introduction

A SPACECRAFT in a geostationary orbit in equinox experiences total solar eclipse for 72 min during an orbital period of 24 h. It seems necessary to examine the effect of solar eclipse on the proper functioning of the components of the spacecraft because during solar eclipse solar heat is not available. Critical design parameters are to be identified, which can be done with the help of thermal sensitivity analysis. Remedial action can be taken thereafter by the thermal designer.

Haftka[1,2] and Haftka and Malkus[3] computed temperature sensitivities for steady-state and transient systems with linear temperature constraints. The direct differentiation (DDF) method and the adjoint variable method were used by House et al.,[4] Tortorelli et al.,[5] and Belegundu[6] for thermal sensitivity calculations. It was pointed out by Belegundu[6] that the final equations obtained in the formulation of the adjoint variable method are identical to those obtained by the Lagrangian (LG) method. Suresha et al.[7] and Suresha and Gupta[8] calculated steady-state thermal sensitivities by the DDF method and the DDF and LG methods, respectively, and compared their computational efficiencies. Hou and Sheen[9] considered discontinuities in design parameters and suggested a numerical method to calculate sensitivities that uses a logical function. In this Note a new method, called here the regularization method, has been suggested to handle discontinuities. This new method is simple and effective.

In a spacecraft, the diode plate is an important component of the shunt regulator from the point of view of dissipating excess solar electrical power. Because of solar eclipses, solar heat input and internal heat generation become discontinuous functions of time. The diode plate has been analyzed here for its temperature sensitivities with respect to different design parameters such as internal heat dissipation, optical properties of the optical solar reflector (OSR), emittance of black paint, and emittance of low-emittance tape.

Problem Formulation

A thermal system of the spacecraft is made up of a large number of components, referred to here as solid elements. The heat transport within a typical mth element can be described by the Fourier heat conduction equation as in Eq. (1).[7] The boundary conditions on the surface of this element can be written as in Eq. (2).[7]

If heat transport among all of the elements is considered using the Fourier heat equation, with appropriate boundary conditions on the surfaces, then the computations become extremely difficult, and a numerical solution may not be possible. However, a lumped-parameter approach[7] in conjunction with finite difference methods[7,8] can be used to compute thermal sensitivities. If a lumped-parameter approach is used, then the following system of equations[7] is obtained:

$$m_i c_{p_i} \frac{\mathrm{d}T_i}{\mathrm{d}t} = \sum_{j=1}^{N} \frac{K_{i,j} A_{c_{i,j}}}{l_{i,j}} (T_j - T_i) + \sigma \sum_{j=1}^{N} \varepsilon_{i,j} F_{i,j} A_{s_i} \left(T_j^4 - T_i^4\right)$$
$$- \sigma \varepsilon_i F_{i,\mathrm{sp}} A_{s_i} T_i^4 + g_i(t) + \alpha_i A_i s \cos\theta$$
$$i = 1, 2, 3, \ldots, N \quad (1)$$

Because of solar eclipses, the internal heat generation term $g_i(t)$ and the last term in Eq. (1), which represents incident solar energy, are discontinuous functions of time.

Equation (1) can be written in compact form as

$$C_i \frac{\mathrm{d}T_i}{\mathrm{d}t} = f_i(T_1, T_2, \ldots, T_N;\ \beta_1, \beta_2, \ldots, \beta_M;\ t)$$
$$i = 1, 2, \ldots, N \quad (2)$$

When Eq. (2) is differentiated with respect to the parameter β_J, $J = 1, 2, \ldots, M$, the following system of first-order differential equations is obtained that is suitable for sensitivity calculations:

$$C_i \frac{\mathrm{d}S_i}{\mathrm{d}t} = \sum_{j=1}^{N} \frac{\partial f_i}{\partial T_j} S_j + \frac{\partial f_i}{\partial \beta_J}$$
$$i = 1, 2, \ldots, N, \quad J = 1, 2, \ldots, M \quad (3)$$

Temperature Constraints

The temperatures of the spacecraft components are to be maintained within some specified range during the life of the spacecraft. For example, the temperature of the diode plate is to be maintained between −40 and 65°C. The mathematical models for the temperature constraints can be written in the following form.

Linear temperature constraints:

$$0 \leq G_i = \frac{T_i - T_{\min}}{T_{\max} - T_{\min}} \leq 1, \qquad i = 1, 2, \ldots, N \tag{4}$$

Nonlinear temperature constraints:

$$0 \leq G_i = \frac{T_i^2 - T_{\min}^2}{T_{\max}^2 - T_{\min}^2} \leq 1, \qquad i = 1, 2, \ldots, N \tag{5}$$

If the temperatures satisfy constraints in Eq. (4), then they also satisfy constraints in Eq. (5) and vice versa. In the DDF method, the preceding two forms of constraints will give the same result but not in the LG method.

Numerical Methods

The DDF method to compute transient sensitivities without discontinuities in the parameters has been discussed earlier.[7] Quasi-steady-state temperatures are first calculated from Eq. (1) and sensitivities from Eq. (3). Details of these computations and definitions of quasi-steady state, thermal sensitivities, and semirelative sensitivity functions may be found in Ref. 7. The extension of the DDF method in the case of discontinuous parameters will be discussed here.

As indicated in Fig. 1, a discontinuous curve in time corresponding to a parameter consists of the lines AB, CD, and EF. For numerical computations, a continuous curve AB′C′D′E′F can be used in place of the discontinuous curve. As the slope of the line B′C′ is increased, the numerical results should converge. Table 1 indicates that the results do converge. However, an extremely small value of λ is not possible because this may destabilize the numerical scheme.

Executional details of the finite element numerical method used by Hou and Sheen[9] were not given. Discontinuities in the parameters were handled by using a logical function. The present finite difference method and its executional details will be discussed here because of their novelty.

If Eq. (3) is multiplied by the row vector φ with N components and is integrated with respect to time from $t = 0$ to t_f, the following equation is obtained:

$$\varphi C \frac{\partial \boldsymbol{T}}{\partial \beta_J}\bigg|_0^{t_f} - \int_0^{t_f} \varphi \frac{\mathrm{d}C}{\mathrm{d}t}\frac{\partial \boldsymbol{T}}{\partial \beta_J}\,\mathrm{d}t - \int_0^{t_f}\left(\frac{\mathrm{d}\varphi}{\mathrm{d}t}C + \varphi\frac{\partial f}{\partial \boldsymbol{T}}\right)\frac{\partial \boldsymbol{T}}{\partial \beta_J}\,\mathrm{d}t = \int_0^{t_f} \varphi \frac{\partial f}{\partial \beta_J}\,\mathrm{d}t \tag{6}$$

It may be noted that some of the terms in Eq. (6) such as $\partial f / \partial \boldsymbol{T}$ refer to differentiation of a row vector with respect to another row vector. It will be assumed that the thermal capacity is independent of time and

$$\varphi(t_f) = 0 \tag{7}$$

$$\frac{\partial \boldsymbol{T}}{\partial \beta_J}\bigg|_{t=0} = 0 \tag{8}$$

Equations (7) and (8) will be used to define the time t_f and the time $t = 0$, respectively. If $t = 0$ is taken as the time at which the integration of Eq. (1) starts to obtain quasi-steady-state temperatures, then Eq. (8) holds. By the use of Eqs. (7) and (8), the first two terms on the left-hand side of Eq. (6) become zero.

In the present problem the temperature constraints G_i, $i = 1, 2, \ldots, N$ are independent of the design parameters, and G_i for any i is a function of only one temperature T_i. Therefore,

$$\frac{\mathrm{d}G_i}{\mathrm{d}\beta_J} = \frac{\partial G_i}{\partial T_i}\frac{\partial T_i}{\partial \beta_J}, \qquad i = 1, 2, \ldots, N, \quad J = 1, 2, \ldots, M \tag{9}$$

If the differential equation to determine φ is taken in the form

$$C^l\left(\frac{\mathrm{d}\varphi}{\mathrm{d}t}\right)^l + \left(\frac{\partial f}{\partial \boldsymbol{T}}\right)^l \varphi^l = \left(\frac{\partial G_i}{\partial \boldsymbol{T}}\right)^l \tag{10}$$

then it is easy to see that

$$\frac{\mathrm{d}G_i}{\mathrm{d}\beta_J} = -\varphi\frac{\partial f}{\partial \beta_J} \tag{11}$$

Table 1 Temperature variation of the diode plate with different values of λ

λ, h	Temperature, °C
0.2	−16.60
0.05	−25.10
0.03	−26.50
0.01	−26.80
0.003	−26.80

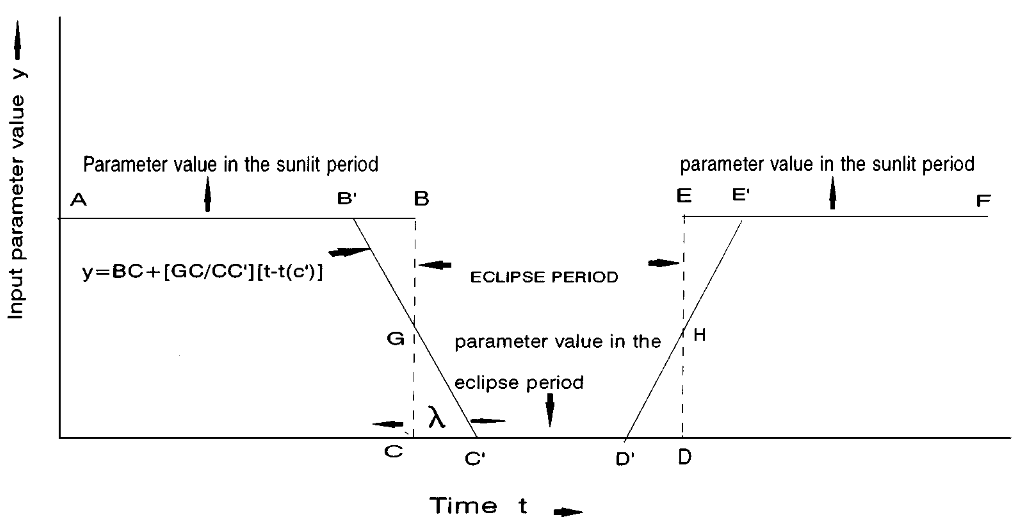

Fig. 1 Regularization of discontinuous heat input parameter.

It may be noted that the quantities $\partial f / \partial \boldsymbol{T}$, $\partial f / \partial \beta_J$, and $\partial G_i / \partial T_i$ are to be calculated analytically. Once φ is determined from Eq. (10), temperature sensitivities can be calculated from Eq. (9).

Definition of t_f

To determine φ, Eq. (10) is to be integrated backward in time with the initial condition taken as $\varphi(t_f) = 0$. It can be proved that if in a quasi-steady state t_f is taken as the time at the end of an orbital period then at the beginning of the orbital period φ also will be zero. In such a case, Eq. (10) cannot be solved uniquely. Suppose quasi-steady state is attained at time, for example, $t = t_s$; then t_f can be taken three or four orbits after t_s so that when backward integration of Eq. (10) is done, φ attains quasi-steady state at time $t = t_s$. Generally quasi-steady state is attained after successive integrations over three or four orbital periods. When t_f was defined as just stated, the results by the DDF method and the LG method agree reasonably well in the sunlit period. During the eclipse, a maximum difference of 37.1% was observed in the semirelatives calculated by these two methods.

Numerical Solution and Discussion of Results

In both the DDF and the LG methods, a shunt regulator is divided into 25 nodes, with 1 node at the diode plate. Equation (1) is to be solved first to obtain quasi-steady-state temperatures by the numerical methods reported in Ref. 7. In Euler's method, the time step was taken as 100 s during the sunlit period and 10 s during the eclipse period. In the Runge–Kutta–Fehlberg method,[7] the subroutine automatically adjusts the time step. In the LG method, derivatives of the constraints are required, and in the transient problems, the functional form of the constraints can make a difference. The backward numerical integration of Eq. (10) may also be done as the forward integration of Eq. (3). The correctness of thermal sensitivities can be checked by satisfying Eq. (12).[8] The DDF method was found to be more accurate than the LG method, at least in the eclipse period. However, the LG method is economical in the present problem because thermal sensitivities are to be obtained at a single node consisting of a diode plate.

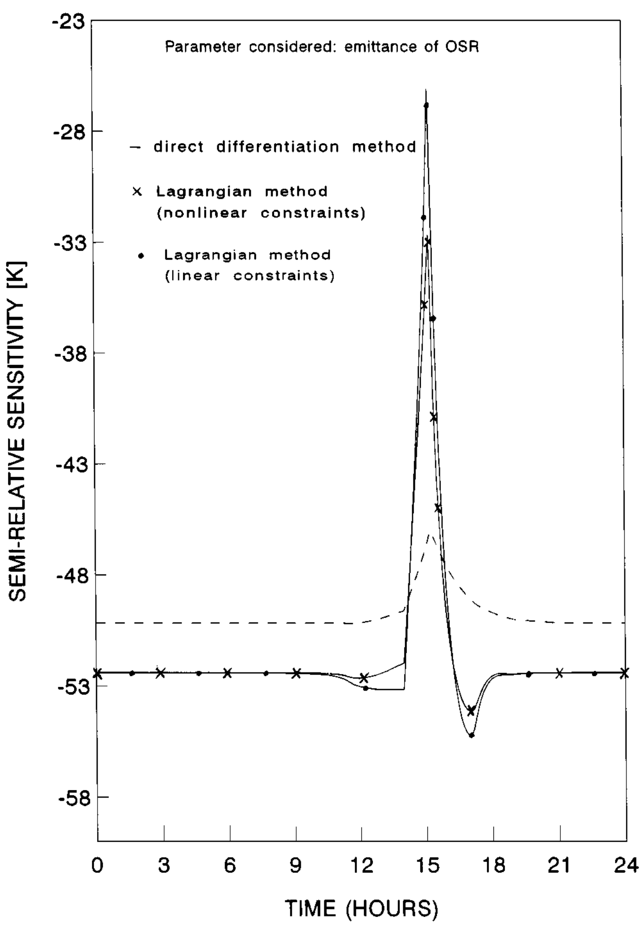

Fig. 2 Semirelative sensitivity of the diode plate with respect to emittance of OSR.

Table 2 Nominal values of design parameters

Parameter	Absorptance	Emittance	Internal heat dissipation, W: Sunlit	Internal heat dissipation, W: Eclipse
OSR diode plate	0.08	0.78	18.90	0.00
Black paint	0.90	0.90		
Low-emittance tape	0.15	0.05		

To do the sensitivity analysis, semirelative functions corresponding to all of the design parameter values given in Table 2 were computed, but to save space only Fig. 2 is being given here for emittance of OSR. Although the sensitivity analysis suggests that power dissipation by the diode plate has the maximum effect on the temperature of the diode plate, due to system constraints this parameter cannot be controlled by the thermal designer. The next important parameter whose variation affects the diode plate temperature most was found to be the emittance of OSR. Because heat input parameters are discontinuous, the effect of any change in the parameter values has to be analyzed in both the sunlit and eclipse periods.

Conclusions

1) A regularization technique suitable for computations with discontinuous design parameters has been suggested.

2) An appropriate procedure to calculate transient thermal sensitivities by the LG method that gives stable results has been indicated. Differences in the results obtained by the DDF and the LG methods were noticed, but they are not as high as reported by Hou and Sheen,[9] who used a logical function to handle discontinuities in the parameters.

3) In the LG method, derivatives of the constraints with respect to temperatures are required. It was found that numerical results may differ in a transient problem if a different functional form of the constraint is considered.

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